by the vapor flow near the bottom of the cavity. However, if such a solution does exist, the details of the flow in this region will prove to have a weak effect on the main vapor flow characteristics far from the bottom, as has been confirmed by calculation.

Numerical study of the model in the supersonic vapor flow regime revealed that no set of external parameters could be found which would provide a cavity with $h/r_0 > 1$. General analysis of the results for both supersonic and infrasonic regimes indicated the great sensitivity of the entire gasdynamic picture to the method of defining surface temperature T_S, which will require more accurate calculation of heat transfer from the surface.

The numerical studies of the proposed model performed, which considered vapor gasdynamics in the quasi-one-dimensional approximation as well as heat loss into the liquid, revealed the following. The model predicts daggerlike melting for materials with sufficiently low thermal conductivity and external pressures ensuring slow escape of the vapor from the cavity. Within the cavity significant regions may exist where equilibrium is maintained not by evaporation, as in [2, 3], but by condensation of vapor on the wall. The channel width is then greater than the diameter of the light beam, and the greater part of the beam energy remains in the liquid. To describe deep penetration of a laser beam into metals, it is obviously necessary to consider the multipath character of vapor flow in the cavity, i.e., to solve the two-dimensional problem.

LITERATURE CITED

1. V. A. Antonov and G. I. Kozlov, "Stationary laser cavity produced by interaction of high-power CO_2 laser radiation with a metal and liquid," Kvantovaya Elektron., 4, No. 8 (1977).
2. J. G. Anderews and D. R. Atthey, "Hydrodynamic limit to penetration of a material by a high-power beam," J. Phys. D: Appl. Phys., 9, No. 15 (1976).
3. A. A. Uglov, V. V. Ivanov, and V. N. Koren'kov, "Calculating the profile of the cavity formed by action of a concentrated heat source," Fiz.-Khim. Obrab. Mater., No. 3 (1977).
4. S. I. Anisimov, Ya. I. Imas, G. S. Romanov, and Yu. V. Khodyko, Action of High Power Radiation on Metals [in Russian], Nauka, Moscow (1970).

PARAMETRIC STUDY OF HYPERSONIC BODY SHAPES

V. A. Bordyug, Yu. A. Vedernikov,
V. G. Dulov, A. I. Shvets,
and V. A. Shchepanovskii

UDC 533.6.013.12

Although the problem of the design of axisymmetric bodies for minimum drag is practically solved, the optimization of three-dimensional aerodynamic shapes still requires detailed analysis. Many years of experience on the design of optimal spatial configurations using some exact solutions [1-5] and approximate specification of aerodynamic load on the body surface [5-12] revealed the need for systemization of experimental results. The first numerical computations of the flow around linear forms [12] only emphasized the need for conducting parametric experiments on detailed flow characteristics around three-dimensional bodies. The effect of aspect ratio and midsection of a star-shaped body with sharp and blunt leading edges on its drag is considered here as a supplement to the computed results [11, 15] and experimental data [16]. It is shown experimeantally that, as revealed by numerical optimization [11, 15], there is a strong dependence of relative drag reduction of stars on their aspect ratio. It is also shown that even for asymmetric star-shaped bodies [17] the critical parameter is the ratio of the circle inscribed at the midsection and the circle circumscribed near it. Slightly blunt leading edges have practically no effect on the general nature of the relations. Besides, a comparison is made, on the basis of approximations, with experimental and exact computational results [18].

1. One of the features of star-shaped configurations is that among their many types there are some for which it is possible to obtain an exact solution for the inviscid flow field, taking into consideration all complex

Novosibirsk. Krasnoyarsk. Moscow. Translated from Zhurnal Priladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 1, pp. 51-57, January-February, 1983. Original article submitted March 18, 1982.

0021-8944/83/2401-0044$07.50

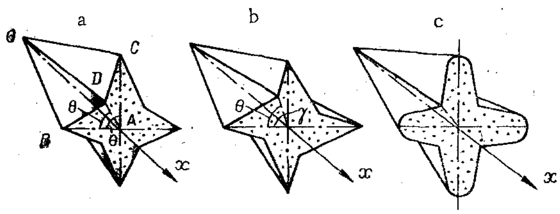

Fig. 1

nonlinear interactions. If the angle of inclination of the leading edge plane σ to the axis and that of the inner edge δ to the axis (Fig. 1a), and the free stream Mach number M_∞ are related by the equation

$$\operatorname{tg}\delta = \operatorname{ctg}\sigma \frac{\sin^2\sigma - 1/M_\infty^2}{\frac{\varkappa+1}{2} - \left(\sin^2\sigma - 1/M_\infty^2\right)} \tag{1.1}$$

(here $\varkappa$ is the adiabatic index), then the flow field in the neighborhood of the right star can be described exactly. The leading edge shock consists of two planes connecting the sharp leading edges. The flow behind the shock is uniform and corresponds to the flow behind plane oblique shock inclined at an angle σ to the free stream at Mach number M_∞. If the values of M_∞, σ, and δ do not satisfy (1.1), then a simple analytical solution for the star-shaped configuration does not exist. Nevertheless, this flow condition could be used as the starting point for the construction of approximate solutions, the estimation of the accuracy of approximate methods, and for the determination of the limits of applicability of empirical theories.

Let the star parameters (Fig. 1a) satisfy (1.1), then the wave drag can be computed exactly:

$$C_p = 2\sin\sigma\sin\delta/\cos(\sigma - \delta) \tag{1.2}$$

(the drag coefficient is based on the dynamic pressure $q = \rho_\infty v_\infty^2/2$ and the midsection area).

Since the surfaces of the body are at the same angle to the free stream, the wave drag coefficient determined by the method of expansion waves and the tangent-wedge method will be constant on the surface and equal to the exact value (1.2). Computations based on Newton's formula gives $C_{pH} = 2\sin^2\delta$ and the relative error in this case is given by $\Delta = \cot\delta \cdot \tan(\sigma - \delta)$, where the quantity $(\sigma - \delta)$ is sufficiently small at large M_∞, since $(\sigma - \delta)$ physically determines the relative thickness of the shock layer. If the modified Newton's equation is used, then $C_{pH} = k\sin^2\delta$, where the coefficient k is determined from the exact or known experimental value of C_p at a certain point on the body surface. For the present configuration $k = 2\sin\sigma/(\cos(\sigma - \delta)\sin\delta)$ and the modified Newton equation give exact solution (1.2).

Comparative analysis of the results of the principal approximate methods with the exact value of the wave drag coefficients for star-shaped configurations show that Newton's method underestimates for real pressure distribution along the surface and the tangent-wedge method overestimates. We note that the real pressure distribution could attain its limiting values.

Figure 1b shows a star-shaped body in a supersonic flow at zero angle of attack, having a nonzero moment relative to the longitudinal axis [17]; γ, θ are the adjacent angles between planes passing through the longitudinal axis and the neighboring outer and inner edges. It is possible to show that for $n \geq 5$ (n is the number of star lobes) and certain relations between geometric parameters of the star and the free stream Mach number, there is a simple analytical solution for the flow around the configuration of the type shown in Fig. 1b. For $n = 4$ (in Fig. 1b this case corresponds to $\gamma = 0$) the solution of the flow problem with plane shock wave and uniform flow behind it does not exist for any geometric parameters at all free stream Mach numbers. The wave drag coefficient for this configuration is always less than the limiting value given by the tangent-wedge method. All conclusions are true for configurations and free stream Mach numbers when the shock is attached to the leading edges. Otherwise the method of tangent – wedge and expansion waves are not applicable.

It is not possible to use the exact solutions for the study of the flow around star-shaped configurations with all spatially permissible parameters. Gasdynamic approach is also not applicable in the case of blunt leading edge bodies. Experimental methods are the principal method of investigation in the case of arbitrary geometric parameters and flow conditions. Approximate computations, as shown above, give an overestimate of gasdynamic parameters and the existence of exact solution for a given combination of parameters is the control point.

2. Closed star-shaped pyramid for symmetric configuration is obtained when $\theta = \pi/n$ (Fig. 1a) and a

configuration with a rotation (Fig. 1b) when $\theta + \gamma = 2\pi/n$. The pressure distribution for the flow around such a body is determined by the equation:

$$p = \frac{2}{\varkappa M_\infty^2} \text{ on } DAC, DAB; \; p = \frac{2}{\varkappa M_\infty^2} + \frac{\sin\sigma \sin\delta}{\cos(\sigma - \delta)} \text{ on } OBDC.$$

Here the pressure coefficient is based on the dynamic pressure $q = \rho_\infty U_\infty^2/2$, where ρ_∞ and p_∞ at the base, stream density and velocity, respectively.

Considering that the pressure $p = p_\infty$ at the base, the wave drag X_V is determined by the relation

$$X_V = 2n \frac{\sin\sigma \sin\delta}{\cos(\sigma - \delta)} \operatorname{tg}\delta \operatorname{tg}\sigma [\operatorname{tg}\theta + \operatorname{tg}(\alpha_0 - \theta)],$$

where $\alpha_0 = 2\pi/n$, $\alpha_0/2 \leq \theta \leq \alpha_0$, $n = 3, 4, \ldots$ are the number of star lobes. After simple transformations we get

$$X_V = 2n \frac{\sin\sigma \sin\delta}{\cos(\sigma - \delta)} \operatorname{tg}\delta \operatorname{tg}\sigma \frac{\sin\alpha_0}{\cos\theta \cos(\alpha_0 - \theta)}.$$

For the limiting case $n \to \infty$ we get $\sin\alpha_0 \to 2\pi/n$, $\cos\theta \to 1$, $\cos(\alpha_0 - \theta) \to 1$, and finally,

$$X_V^{\lim} = 4\pi \frac{\sin^2\sigma \sin^2\delta}{\cos\sigma \cos\delta \cos(\sigma - \delta)}.$$

Wave drag approaches a certain limiting value determined by the free stream parameters with unlimited increase in the number of star lobes.

In order to determine the skin-friction drag the surface area S is estimated:

$$S \geqslant n[\sqrt{(\operatorname{tg}\sigma - \operatorname{tg}\delta)^2 + (\operatorname{tg}\sigma \operatorname{tg}\theta)^2} + \sqrt{(\operatorname{tg}\sigma - \operatorname{tg}\delta)^2 + (\operatorname{tg}\sigma \operatorname{tg}(\alpha_0 - \theta))^2}] \geqslant 2n\sqrt{(\operatorname{tg}\sigma - \operatorname{tg}\delta)^2 + (\operatorname{tg}\sigma \operatorname{tg}(\alpha_0 - \theta))^2} = S(n).$$

It is seen that as $n \to \infty$, $S(n) \to \infty$. This indicates that $X_{tr} = fS$ (f is the average drag coefficient) and unboundedly increases in the limiting case. The latter indicates the presence of optimum number of lobes n for which the total drag is a minimum. This result is confirmed in [15, 16]. It remains to determine the effect of aspect ratio on star-shaped body and the shape of its midsection.

3. Plane star-shaped bodies with sharp leading edges and symmetric midsection are investigated (Fig. 1a). The ratio of the diameter of the circle inscribed in the star-shaped midsection of a cone of equal cross-sectional area d_{vp}/d_e equals 0.5. The number of cycles of the star, taking into account the results of parametric optimization [15, 16], is chosen to be four. Bodies considered are those with aspect ratios of equivalent cone from one to three. Apart from a cone, a stepped body of revolution, optimized for each aspect ratio, has also been considered. Experiments on these configurations were conducted in the wind-tunnel T-313 ITPM SO AN SSSR for $M = 3$; 4; 5; and $Re_{1M} \approx 10^7$ on models with the diameter of equivalent axisymmetric midsection of 65 mm.

The solid line 2 in Fig. 2 is the computed relation of the ratio of the drag of the equivalent cone X_k to the drag $X_{o.t}$ of curvilinear star shaped body with four lobes, optimally designed for different aspect ratios λ of the cone in [11, 15]. The dash–dot line 3 shows similar dependence for stepped axisymmetric body with minimum drag. Experimental relation $X_k/X_{o.t} = f(\lambda)$ for plane star-shaped body and optimum stepped body of revolution obtained for $M = 4$ are shown, respectively, by dashed curve 4 and 5 (computed curve 1 is obtained for optimization without the singularities of Newtonian approximation [11]).

The qualitative agreement of the nature of computed and experimental curves is apparent. The quantitative disagreement of curves for star-shaped bodies of small aspect ratio is caused mainly by the difference in the equivalence used in computation and experiment. For the sake of convenience of the intermediate computations in optimization [11, 15], a cone inscribed in the star-shaped body was the equivalent body. In view of this, at small aspect ratios, whenever large spans of stars are designed, the inscribed cones differ mainly from cones with equally large midsections. Taking all these facts into consideration, the computed curve is slightly raised in the small aspect ratio region.

At aspect ratios close to three, the difference between computed and experimental results can be, apparently, explained by the different skin-friction contribution to the total drag of the respective star-shaped bodies.

The Newtonian approximation for axisymmetric bodies agree satisfactorily with experimental results for all the aspect ratios studied (see curves 3 and 5).

As a result of the comparative analysis carried out, it is possible to rely on the optimized results for

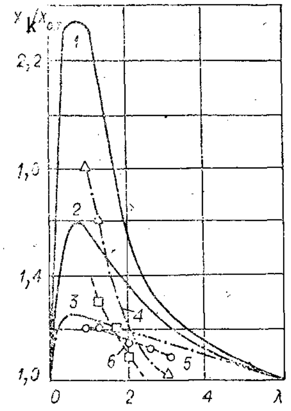

Fig. 2

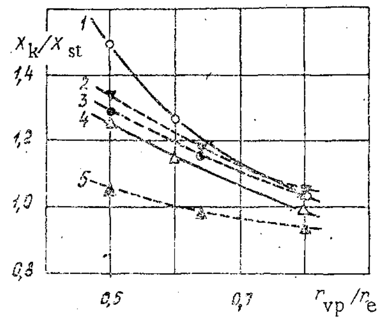

Fig. 3

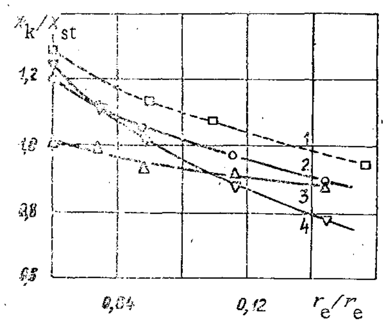

Fig. 4

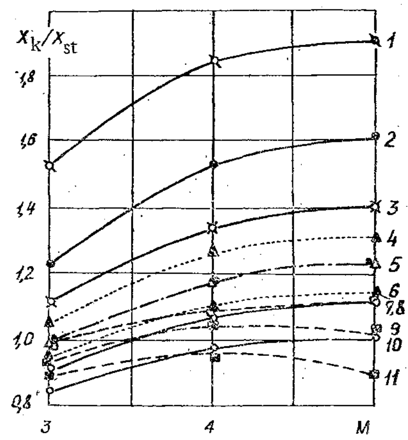

Fig. 5

three-dimensional configurations in describing the aerodynamic loading with Newton's shock theory.

4. Plane star-shaped bodies with sharp leading edges and four-lobed midsection with rotation at angle $\gamma = 0$ (Fig. 1b) are investigated experimentally. The ratios of the diameter of the circle inscribed in the middle of the star to the diameter of the midsection of an equivalent cone with aspect ratio 1.3 are chosen equal to 0.57, 0.64, 0.8, with aspect ratios 1.6; 2 the ratio equals 0.57. Besides the cone, for an aspect ratio λ = 1.3, a symmetric star-shaped body with the same ratios of (d_{vp}/d_e) has been used as an equivalent.

The dependence of the relative drag X_k/X_{st} of the star-shaped body with rotation on the aspect ratio λ for M = 4 and $r_{vp}/r_e = 0.57$ is given in Fig. 2 by the curve 6 which is practically equidistant from the curve 4 for symmetric stars. This shows that it is possible to optimize, using the approach [11, 15], symmetric stars and stars with rotation, by appropriately coupling the cyclical surfaces during the computation.

The dependence of X_k/X_{st} at zero angle of attach on the ratio r_{vp}/r_e for aspect ratio $\lambda = 1.3$ for symmetric stars (solid lines) and stars with rotation are shown in Fig. 3. The curve 2 corresponds to M = 5; 1, 3 are for m = 4; 4, 5 for M = 3. It is seen that, as in [16], the parameter r_{vp}/r_e is critical for the reduction of total aerodynamic drag of star-shaped bodies. Here, because of differences in the values of the integral angles of attack of windward surfaces the star-shaped bodies with different midsection shapes have different drag for one and the same value of r_{vp}/r_e.

5. Plane star-shaped bodies with four lobed midsection and blunt leading edges (Fig. 1c) are investigated. The ratios of the diameter of the circle inscribed in the star-shaped midsection to the diameter of the midsection of an equivalent cone with aspect ratios 1.46 and 1.83 are equated, respectively, to 0.66 and 0.525. For an aspect ratio 1.46 the ratios of the diameter of cylindrical blunting to the diameter of equivalent cone are chosen to be equal to 0, 0.046, 0.095, 0.187, and 0.282. For an aspect ratio 1.83 these ratios are respectively 0, 0.0282, 0.0563, 0.112, 0.169. Besides sharp cone for blunt star with parameter $\lambda = 1.43$ and $r_z/r_e = 0.29$, a blunt cone with $r_z/r_e = 0.11$ has been constructed as an equivalent in terms of cross-sectional areas.

Figure 4 shows the dependence of the ratios of drag of the basic sharp cone X_k and blunt stars X_{st} on the ratio of radii of blunt leading edges r_z and base of the cone r_e. The curve 1 corresponds to the drag of a blunt star with an aspect ratio $\lambda = 1.43$ at $M = 4$.

Curves 2-4 ($M = 4, 3, 5$, respectively) correspond to blunt star with aspect ratio $\lambda = 1.85$.

Comparing the practically equivalent curves 1 and 2, it is possible to conclude on the identity of the influence of cylindrical bluntness of leading edges on the drag of stars with intervals in aspect ratios from 1.3 to 2.0.

From a comparison of the curves 2, 3, and 4 it is seen that there is a nonmonotonic effect of the Mach number on the drag of star-shaped bodies with cylindrical blunt leading edges. A gain in drag when compared to the basic sharp cone is observed in this case only for $M = 4, 5$ with $r_z/r_e \leq 0.06$.

It is significant that for maximum bluntness ($r_z/r_e = 0.29$) the star with aspect ratio $\lambda = 1.43$ at $M = 4$ loses in terms of drag to the equivalent blunt cone by only 5%.

6. The resultant relations between relative drag X_k/X_{st} and Mach number for three variants of star-shaped bodies (see Fig. 1) are given in Fig. 5. The solid lines correspond to symmetric stars, dotted curves represent stars with rotation, and dashed curves are for stars with blunt leading edges. The dash–dot curve 5 refers to symmetric star ($\lambda = 2$) as well as star with rotation ($\lambda = 1.6$). The curves 1-4, 6, 8, and 10 correspond to aspect ratios $\lambda = 1$, 1.3, 1.6, 1.3, 2, 2.4, 2.8, curves 7, 9, and 11 correspond to aspect ratio $\lambda = 1.85$ and relative radii of bluntness $r_z/r_e = 0.028$, 0.056, 0.112.

It is seen from the above plots that for star-shaped bodies with sharp leading edges, with increase in Mach number the advantage with respect to the drag of equivalent cone improves in the entire range of aspect ratios under study. For stars with blunt leading edges the relations $X_k/X_{st} = f(M)$ have points of inflection. Here the larger value of r_z/r_e corresponds to lower value of Mach number at which the optimum is achieved.

It is worth observing that only the curves 10, 11 have a value less than 1 in the entire range of parameters. The remaining nine curves exhibit appreciable advantage of star shaped bodies compared to cones with equivalent cross-sectional areas.

Thus, the above parametric study indicates the correctness of the general approach [11, 15] to the optimization of hypersonic three-dimensional bodies with symmetric star-shaped midsections and stars with rotation. This approach makes it possible, apparently, to introduce into the search for optimal solution a small leading edge bluntness [19, 20] as an "external" (specified) variable parameter, where a combination of aerodynamic and geometric characteristics can be used as objective functions.

LITERATURE CITED

1. G. I. Maipar, "Wave drag of nonaxisymmetric bodies at supersonic speeds," Prikl. Mat. Mekh., 23, No. 2 (1959).
2. T. R. F. Nonweiler, "Aerodynamic problems of manned space vehicles," J. R. Soc., 63, No. 585 (1959).
3. A. L. Gonor, "Exact solution for some three-dimensional bodies in supersonic flow," Prikl. Mat. Mekh., 28, No. 5 (1964).
4. V. V. Kel'dysh and G. I. Maikapar, "Gasdynamic design of hypersonic aircraft," Izv. Akad. Nauk SSSR, Mekh. Zhikd. Gaza, No. 3 (1969).
5. V. G. Dulov, "Certain formulations of three-dimensional optimization problems in hypersonic aerodynamics," Zh. Prikl. Mekh. Tekh. Fiz., No. 5 (1976).
6. A. Ferry, N. Ness, and R. Kaplita, "Supersonic flow over conical bodies without axial symmetry," J. Aero. Sci., 20, No.8 (1953).
7. G. G. Chernyi and A. L. Gonor, "The determination of body minimum drag using the Newton and Busemann pressure laws," in: Paper presented at the symposium on Extremal Problems in Aerodynamics, Boeing Scientific-Research Laboratories, Seattle, Washington (1962).
8. A. Miele and G. R. Saaris, "On the optimum transversal contour of a body at hypersonic speeds," Astronautica Acta, 9, No. 3 (1963).
9. A. L. Gonor, "Determination of the shape of optimum body taking skin-friction into consideration," Izv. Akad. Nauk SSSR, Mekh. Mashinostr., No. 4 (1965).
10. V. V. Kravets, "Comparative characteristics of aerodynamic coefficients of half-wedges and body of revolution with equivalent volume and length," Hydroaeromechanics and Theory of Elasticity, 2, (1970).

11. Yu. A. Vedernikov, V. G. Dulov, and A. F. Latypov, "Optimization of hypersonic body shapes," Zh. Prikl. Mekh. Tekh. Fiz., No. 1 (1979).
12. A. A. Gusarov, V. M. Dvoretskii, et al., "Theoretical and experimental study of the aerodynamic characteristics of bodies," Izv. Akad. Nauk SSSR, Mekh. Zhidk. Gaza, No. 1 (1979).
13. A. L. Gonor, M. N. Kazakov, et al., "Aerodynamic coefficients of nonconical bodies at supersonic speeds," Izv. Akad. Nauk SSSR, Mekh. Zhidk. Gaza, No. 1 (1971).
14. M. N. Kazakov, V. V. Kravets, and A. I. Shvets, "Aerodynamic coefficients of nonconical bodies with star-shaped cross sections," Izv. Akad. Nauk SSSR, Mekh. Zhidk. Gaza, No. 6 (1974).
15. Yu. A. Vedernikov, V. G. Dulov, and A. F. Latypov, "Three-dimensional hypersonic configurations with minimum drag," in: Studies on Hypersonic Aerodynamics, Izd. Institute of Theoretical and Applied Mechanics, Siberian Branch, Academy of Sciences of the USSR and the Computer Center, Siberian Branch, Academy of Sciences of the USSR, Novosibirsk—Krasnoyarsk (1978).
16. Yu. A. Vedernikov, A. L. Gonor, et al., "Aerodynamic characteristics of star-shaped bodies at M = 3-5," Izv. Akad. Nauk SSSR, Mekh. Zhidk. Gaza, No. 4 (1981).
17. G. I. Maikapar, "Bodies formed by stream surfaces of conical flows," Izv. Akad. Nauk SSSR, Mekh. Zhidk. Gaza, No. 1 (1966).
18. V. A. Shchepanovskii, "Construction of three-dimensional flow from plane flows," Izv. Akad. Nauk SSSR, Ser. Tekh. Nauk, 2, No. 8 (1972).
19. V. V. Kel'dysh, "Drag of arrow-shaped blunt leading edge wing at hypersonic speeds," Izv. Akad. Nauk SSSR, Mekh. Zhidk. Gaza, No. 5 (1964).
20. G. G. Chernyi, Fluid Flow at High Supersonic Speeds [in Russian], Fizmatgiz, Moscow (1959).

FLAT SUPERSONIC UNDEREXPANDED JETS USING A LASER SCHLIEREN METHOD

V. A. Kochnev and I. M. Naboko

UDC 533.6.011.5:621.375.826

1. Interest in flat jet flows arose due to progress in nonequilibrium physicochemical kinetics, creation of gasdynamic lasers (GDL), and solution of other problems arising in the new technology. In particular, in modeling GDL, flat jets have certain advantages over the usual nozzles: for example, they provide maximum expansion velocity of the flow with planar geometry. However, the interrelation of the kinetic and gasdynamic processes occurring in the jets, as well as the presence of viscous effects, manifested, for example, in the formation of boundary layers, complicate the study of supersonic, high-enthalpy, gas jets and require that these jets be experimentally studied.

The purpose of this work is to investigate gasdynamic characteristics of a flat jet: experimental determination of the density profile along the center of the stream tube of the jet and numerical estimates of the boundary layer, arising on the lateral surfaces bounding the jet, based on a theoretical analysis. The proposed experimental method, which has high sensitivity and temporal resolution $\leq 1\,\mu$sec, is based on measuring a sequence of density gradients, relating to different cross sections of the flow studied with the help of the laser schlieren method [1].

There are several papers concerned with investigation of flat jets flowing out of a sonic slit nozzle into a space bounded by two parallel surfaces [2-5]. The wave structure of such a stationary flow was studied by the shadow method in [2-4]. A generalizing dependence of the location of the central jump as a function of the determining parameters was obtained in [4]. The results are compared with data for axisymmetrical jets. In some regimes, separation of the boundary layer, forming on the lateral surfaces bounding the jet, is observed [2]. The flow field of a flat, weakly underexpanded perfect gas jet is calculated in [5] using the stabilization method.

From an analysis of the theoretical model both for flat and for axisymmetrical jets, it is possible to obtain a generalized relation for the density distribution at the center of the stream tube [3]:

Moscow. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 1, pp. 57-65, January-February, 1983. Original article submitted June 26, 1981.